Breakdown of Hydrodynamic Transport Theory in the Ordered Phase of Helimagnets

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It is shown that strong fluctuations preclude a hydrodynamic description of transport phenomena in helimagnets, such as MnSi, at $T > 0$. This breakdown of hydrodynamics is analogous to the one in chiral liquid crystals. Mode-mode coupling effects lead to infinite renormalizations of various transport coefficients, and the actual macroscopic description is nonlocal. At $T = 0$ these effects are weakened due to the fluctuation-dissipation theorem, and the renormalizations remain finite. Observable consequences of these results, as manifested in the neutron scattering cross-section, are discussed.

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Soft modes, or excitations at low frequencies and small wave numbers, in many-body systems are crucial for the behavior at long distances and times. They also lead to characteristic nonanalyticities in the temperature (T) dependence of observables that are given by integrals over the excitation spectrum, such as the specific heat. Examples include shear diffusion in classical fluids [1], the 'diffuson' in quenched disordered electron systems at $T = 0$ [2], magnons in magnetically ordered solids [3], the director fluctuations in liquid crystals [4], and many others. Interactions between such soft modes, often referred to as mode-mode coupling effects, lead to even more interesting phenomena. In classical fluids, they produce the so-called long-time tails, or non-exponential decay of time correlation functions, e.g., the transverse velocity autocorrelation function [5]. In smectic and cholesteric liquid crystals they are even stronger, and lead to a breakdown of hydrodynamics, with certain viscosities diverging for vanishing frequency ω as $1/\omega$ [6]. The weak-localization anomalies in disordered electron systems [2] belong to the same category of effects, although they are usually not cast in this language. Technically, the mode-mode coupling effects take the form of nonlinearities in dynamical equations, or loops in a field-theoretic description.

The above examples make it clear that mode-mode coupling effects, and the singularities induced by them, are relevant for both classical and quantum many-body systems. An obvious question is therefore what happens to the classical singularities as $T \to 0$. From the structure of the classical singularities it is likely that they become weaker with decreasing temperature, while quantum singularities may develop that become stronger, but this has never been investigated.

One purpose of the present Letter is to study an explicit example of what happens to classical mode-mode coupling singularities as $T \to 0$. Our example are helimagnets, such as MnSi, which have recently been shown to exhibit interesting analogies with smectic and cholesteric liquid crystals. These analogies exist because the order parameters in the two systems are essentially identical and the symmetries of the respective coarse-grained free-energy functionals are closely related. Consequently, a tree-level analysis of both a phenomenological and a microscopic theory of helimagnets [7] reveals a Goldstone mode, and its manifestation in observables, that are very similar to the corresponding properties of chiral liquid crystals [4, 8]. These properties are very unusual because of the anisotropic nature of the helical Goldstone mode, which is softer in the direction transverse to the pitch vector of the helix than in the longitudinal direction.

A second purpose of this Letter is to investigate whether these analogies extend beyond the tree level, i.e., whether the mode-mode coupling effects in helimagnets are as strong as in chiral liquid crystals. We find that this is indeed the case. Some of the most interesting and dramatic effects are the fluctuation renormalizations of the transport coefficients that determine, for example, the line width of the neutron scattering signal due to the helical Goldstone mode. At $T > 0$ we find that fluctuation effects are so strong that they lead to infinite renormalizations of the transport coefficients and elastic constants. Physically, this implies that at long length scales, and for low frequencies, the actual macroscopic description is nonlocal. In the quantum or zero temperature limit we find that the divergencies are removed due to the structure of the fluctuation-dissipation theorem, and the transport coefficients and elastic constants saturate, albeit at a very low temperature. As we discuss below, our predictions will be observable in precision neutron scattering experiments.

Our starting point is a phenomenological hydrodynamic theory [9, 10] for fluctuations in helimagnets. We will take into account the most important nonlinear terms, and will show that the nonlinearities qualitatively affect the physics. Let the mean magnetization $\langle \boldsymbol{m} \rangle$ lie in the x-y plane, with the axis of the helix pointing in

the z-direction:

$$\langle \boldsymbol{m} \rangle(\boldsymbol{x}) = m_0 \, (\cos qz, \sin qz, 0), \tag{1}$$

with q the pitch wave number of the helix. Typically, $q \ll k_{\rm F}$, with $k_{\rm F}$ the Fermi wave number. The fluctuations about this average state are described in terms of a dimensionless generalized phase variable $u(\boldsymbol{x})$, which describes the fluctuations of the x and y-components of $\boldsymbol{m}$, and the z-component of the magnetization, $m_z(\boldsymbol{x}) \equiv m(\boldsymbol{x})$ [7]. The course-grained Hamiltonian for these variables takes the form

$$\begin{aligned} H[u,m] &= \frac{1}{2}\int d\boldsymbol{x} \ \left[aq^2 m^2 - 2hm\right] \\ &+\frac{1}{2}\int d\boldsymbol{x} \ \left[c_z\left(\partial_z u - \frac{1}{2q}\left(\boldsymbol{\nabla} u\right)^2\right)^2 + c_\perp\left(\boldsymbol{\nabla}_\perp^2 u\right)^2\right]. \end{aligned} \tag{2}$$

Here a is the coefficient of the square-gradient term in a purely magnetic Hamiltonian, h is the magnetic field conjugate to m, c_z and $c_\perp$ are elastic constants, and $\boldsymbol{\nabla}_\perp^2 = \partial_x^2 + \partial_y^2$. The first line in Eq. (2) is just the Gaussian part of a Landau Hamiltonian for m; higher powers of m are not important for our purposes. The second line is the effective Hamiltonian for u, which is constrained by symmetry considerations [4, 6]. Translational invariance implies that only gradients of u can enter, while rotational invariance requires that pure rotations of the helix do no cost any energy, which restricts the combinations of gradients that can enter the Hamiltonian. Up to quartic terms in u, Eq. (2) is the most general Hamiltonian consistent with these constraints.

Equation (2) completely describes the thermodynamics of the system via the partition function $Z = {\rm Tr}\,\exp(-H/T)$ (we use units such that $k_{\rm B} = 1$). The dynamics are described by coupled dynamical equations that follow [7] from the usual time-dependent Ginzburg-Landau theory for the magnetization [9] and read

$$\partial_t u = \gamma\,\frac{\delta H}{\delta m} - \Gamma_u'\,\frac{\delta H}{\delta u} + \zeta_u, \tag{3a}$$

$$\partial_t m = -\gamma\,\frac{\delta H}{\delta u} + \left[\Gamma_z'\,\partial_z^2 + \Gamma_\perp'\,\boldsymbol{\nabla}_\perp^2\right]\,\frac{\delta H}{\delta m} + \zeta. \tag{3b}$$

Here γ is a constant, and so are the transport coefficients $\Gamma'_{u,z,\perp}$ [11]. h is the (magnetic) field conjugate to m, and ζ_u and ζ are Langevin forces. In Eq. (3a) the dissipative term $\propto \Gamma_u'$, as well as the Langevin force ζ_u, turn out to be unimportant for the long-wavelength effects considered here, and we will neglect them in what follows.

Consider the magnetic susceptibility or linear response function χ, defined by the response of the average magnetization to an applied magnetic field,

$$\langle m(\boldsymbol{k},\omega)\rangle = \chi(\boldsymbol{k},\omega)\,h(\boldsymbol{k},\omega). \tag{4}$$

The Langevin force ζ is fixed by requiring that the correlation function $F_{mm}(\boldsymbol{k},\omega) = \langle|m(\boldsymbol{k},\omega)|\rangle$ [12] is related to the imaginary part or spectrum of χ, $\chi''(\boldsymbol{k},\omega)$, by the fluctuation-dissipation theorem [13]

$$F_{mm}(\boldsymbol{k},\omega) = \chi''(\boldsymbol{k},\omega)\,\coth(\hbar\omega/2T). \tag{5}$$

The renormalizations of the elastic coefficients and transport coefficients are determined by examining how fluctuations modify the linear response function. To zeroth order in the non-Gaussian terms in Eq. (2) the latter is easily found, from Eqs. (3), to be

$$\chi_0(\boldsymbol{k},\omega) = \frac{\Omega_0^2(\boldsymbol{k}) - i\omega\Gamma_0(\boldsymbol{k})}{aq^2\left[-\omega^2 + \Omega_0^2(\boldsymbol{k}) - i\omega\Gamma_0(\boldsymbol{k})\right]}. \tag{6a}$$

Here

$$\Omega_0(\boldsymbol{k}) = \gamma\,a^{1/2}q\,\left[c_z k_z^2 + c_\perp \boldsymbol{k}_\perp^4\right]^{1/2} \tag{6b}$$

is the helimagnon resonance frequency, and

$$\Gamma_0(\boldsymbol{k}) = \Gamma_z k_z^2 + \Gamma_\perp \boldsymbol{k}_\perp^2, \tag{6c}$$

with $\Gamma_{z,\perp} = aq^2\Gamma'_{z,\perp}$, is a damping coefficient. The spectrum of χ is proportional to the neutron scattering cross-section, and the renormalized counterpart of Γ_0 determines the line width of the helimagnon resonance [13]. The $\langle uu\rangle$ and $\langle mu\rangle$ correlation functions can be related to F_{mm} by Fourier transforming Eq. (3a) to obtain

$$u(\boldsymbol{k},\omega) = \gamma\left[aq^2 m(\boldsymbol{k},\omega) - h\right]/(-i\omega). \tag{7}$$

Loop corrections to χ_0 due to the non-Gaussian terms in Eq. (2), which lead to nonlinearities in the dynamical equations, can be calculated by means of standard perturbation theory for χ. The two one-loop diagrams are shown schematically in Fig. 1. It is convenient to consider

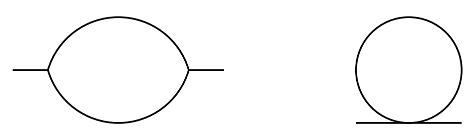

FIG. 1: One-loop contributions to the response function χ. The two diagrams result from the cubic and quartic in u terms, respectively, in Eq. (2).

the renormalization of χ_0^{-1}, whose one-loop correction we denote by $\delta\chi^{-1}$. We obtain

$$\begin{aligned} \delta\chi^{-1}(\boldsymbol{k},\omega) &= \frac{\omega^2}{\left[\Omega_0^2(\boldsymbol{k}) - i\omega\Gamma_0(\boldsymbol{k})\right]^2}\,\left[I_3(\boldsymbol{k},\omega) + I_4(\boldsymbol{k},\omega)\right. \\ &\qquad \left. - C(\boldsymbol{k})\right], \end{aligned} \tag{8}$$

where

$$C(\boldsymbol{k}) = T\Lambda^2\boldsymbol{k}_\perp^2\,\frac{c_z^{1/2}(\gamma aq^2)^2}{16\pi q^2 c_\perp^{1/2}}, \tag{9}$$

and

$$I_3(\boldsymbol{k},\omega) = \frac{c_z^2(\gamma aq^2)^6}{4aq^4}\frac{1}{V}\sum_{\boldsymbol{k}_1} W_3(\boldsymbol{k},\boldsymbol{k}_1)\int_{-\infty}^{\infty}\frac{d\omega_1}{2\pi}\,\frac{F_{mm}(\boldsymbol{k}-\boldsymbol{k}_1,\omega-\omega_1)}{(\omega-\omega_1)^2\,[-\omega_1^2+\Omega_0^2(\boldsymbol{k}_1)-i\omega_1\Gamma_0(\boldsymbol{k}_1)]}\,, \tag{10a}$$

$$I_4(\boldsymbol{k},\omega) = \frac{c_z(\gamma aq^2)^4}{4q^2}\frac{1}{V}\sum_{\boldsymbol{k}_1} W_4(\boldsymbol{k},\boldsymbol{k}_1)\int_{-\infty}^{\infty}\frac{d\omega_1}{2\pi}\,\frac{F_{mm}(\boldsymbol{k}_1,\omega_1)}{\omega_1^2}\,. \tag{10b}$$

Here

$$W_3(\boldsymbol{k},\boldsymbol{p}) = [V_3(\boldsymbol{k},\boldsymbol{p})+V_3(\boldsymbol{k},\boldsymbol{k}-\boldsymbol{p})] \times [V_3(\boldsymbol{p},\boldsymbol{k})+V_3(\boldsymbol{p},\boldsymbol{p}-\boldsymbol{k})]\,, \tag{11a}$$

$$W_4(\boldsymbol{k},\boldsymbol{p}) = V_4(\boldsymbol{k},\boldsymbol{k},\boldsymbol{p})+V_4(\boldsymbol{k},\boldsymbol{p},\boldsymbol{k}) + V_4(\boldsymbol{k},-\boldsymbol{p},\boldsymbol{p}), \tag{11b}$$

with

$$V_3(\boldsymbol{k},\boldsymbol{p}) = (k_z\boldsymbol{p}_\perp + 2p_z\boldsymbol{k}_\perp)\cdot(\boldsymbol{k}_\perp-\boldsymbol{p}_\perp)\,, \tag{11c}$$

$$V_4(\boldsymbol{k},\boldsymbol{p}_1,\boldsymbol{p}_2) = [(\boldsymbol{k}\cdot\boldsymbol{p}_{1\perp})\,\boldsymbol{p}_{2\perp}+(\boldsymbol{p}_{1\perp}\cdot\boldsymbol{p}_{2\perp})\,\boldsymbol{k}_\perp] \cdot(\boldsymbol{p}_{1\perp}+\boldsymbol{p}_{2\perp}-\boldsymbol{k}_\perp)\,. \tag{11d}$$

The first and second terms on the right-hand side of Eq. (8) correspond to the two diagrams shown in Fig. 1. The third term arises from a counterterm that needs to be added to the Hamiltonian in order to ensure that one expands about a proper saddle point, rather than its mean-field approximation [14]. Λ is an ultraviolet cut-off common to all momentum integrals. The technical effect of the counterterm is to subtract the $\boldsymbol{k}_\perp^2$ contributions that arise in perturbation theory and to ensure that the resonance frequency retains the structure given in Eq. (6b), which is dictated by rotational symmetry.

Using Eqs. (5,6) in Eqs. (8) - (11) leads to the leading order fluctuation renormalization of the response function. Note the difference between the classical and quantum cases: In the classical limit, the coth in Eq. (5) contributes a factor $T/(\omega-\omega_1)$ to the integrand in Eq. (10a), while in the quantum limit it contributes a step function. The important implication is that in the quantum limit, the analog of classical mode coupling effects are relatively less singular at low frequencies.

Equation (8) effectively defines a renormalization of the elastic coefficients and the transport coefficients [15]. Denoting the corrections to Ω_0^2 and Γ_0 by $\delta\Omega^2$ and $\delta\Gamma$, respectively, we find

$$\delta\Omega^2(\boldsymbol{k}) = \frac{1}{aq^2}\,\mathrm{Re}\,[I_3(\boldsymbol{k},0)+I_4(\boldsymbol{k},0)-C(\boldsymbol{k})]\,, \tag{12a}$$

$$\delta\Gamma(\boldsymbol{k},\omega) = \frac{-1}{aq^2\omega}\,\mathrm{Im}\,I_3(\boldsymbol{k},\omega). \tag{12b}$$

Equations (12) are the central results of this paper. In the classical limit, $\hbar=0$, at zero external wave number and finite frequency the perturbative corrections to the transport coefficients for small $\Gamma_\perp$ are

$$\delta\Gamma_z = \frac{(\gamma aq^2)^2c_z^{3/2}}{128aq^4c_\perp^{3/2}}\,\frac{T}{|\omega|}\,, \tag{13a}$$

$$\delta\Gamma_\perp = \frac{(\gamma aq^2)^2c_z^{1/2}}{32\pi aq^4c_\perp^{1/2}\Gamma_\perp}\,T\,\ln(\Lambda^2\Gamma_\perp/|\omega|)\,. \tag{13b}$$

For the one-loop corrections to the elastic coefficients at zero frequency we find

$$\delta c_z = -\frac{c_z^{3/2}T}{32\pi q^2c_\perp^{3/2}}\,\ln(\Lambda^2/k_\perp^2)\,, \tag{14a}$$

$$\delta c_\perp = \frac{c_z^{1/2}T}{64\pi q^2c_\perp^{1/2}}\,\ln(\Lambda^2/k_\perp^2)\,. \tag{14b}$$

Equations (14) can also be obtained from equilibrium statistical mechanics considerations, by renormalizing the Hamiltonian, Eq. (2). Note that Γ_z, $\Gamma_\perp$, and $c_\perp$ are renormalized to infinity, while c_z is renormalized to zero [16]. Also note that the fluctuation corrections are all linearly dependent on the temperature. This is due to the fact that all of the excitations are bosonic in character.

Physically, the Eqs. (14) imply that the helimagnon state is, strictly speaking, not stable at finite temperatures. The same is true for certain liquid crystals that show one-dimensional order, and it shows that the analogies between helimagnets and liquid crystals pointed out in Ref. [7] at a Gaussian level do indeed carry over to the respective nonlinear theories. This is a nontrivial observation, since the dynamical equations are different in the two cases. However, practically speaking one must go to enormous length or time scales for these divergencies to dominate the zero-loop effects. In liquid crystals the singularity is typically cut off by finite system size effects; in a helimagnet, it will be cut off by crystal-field effects that break the rotational symmetry. The strongest effect is the renormalization of Γ_z, which shows a linear divergency rather than a logarithmic one.

We now turn to the quantum or zero-temperature limit, where the coth in Eq. (5) turns in to a step function. $\delta\Gamma_\perp$, δc_z, and $\delta c_\perp$ are then finite by power counting. $\delta\Gamma_z$ one might expect to still be logarithmically divergent, but a detailed inspection of the relevant integral shows

that it, too, is finite. We find

$$\delta\Gamma_z = \frac{(\gamma a q^2)^2 c_z^{3/2}}{512\, a q^4 c_\perp^{3/2}} \quad , \quad (T=0)\ . \tag{15}$$

Comparing with Eq. (14a) we see that $\delta\Gamma_z$ saturates at a value corresponding to $T = |\omega|/4$. All of the classical divergencies thus vanish at $T = 0$.

Let us finally provide order-of-magnitude estimates for the magnitude of these fluctuation effects. The coefficients a (which dimensionally is a length squared) and γ^2 (which dimensionally is an energy times a length cubed) have nothing fundamental to do with magnetism and are of order $a \sim 1/k_{\rm F}^2$ and $\gamma^2 \sim \epsilon_{\rm F}/k_{\rm F}^3$, respectively. We denote the Fermi energy, wave number, and velocity by $\epsilon_{\rm F}$, $k_{\rm F}$, and $v_{\rm F}$, respectively, and the Stoner gap or exchange splitting by λ. The tilde indicates that we are leaving out a numerical prefactor that one expects to be close to unity. For the elastic coefficients one expects $c_z \sim c_\perp q^2 \propto \lambda^2/v_{\rm F}$. However, the model calculation of Ref. 7 yielded a small prefactor on the order of 10^{-3} for these coefficients, and we will use $c_z = 2c_\perp q^2 \sim 0.001\,\lambda^2/v_{\rm F}$ for our estimates. The transport coefficients Γ'_z and $\Gamma'_\perp$ have again nothing to do with magnetism, and we expect $\Gamma'_z \sim \Gamma'_\perp \sim v_{\rm F}^2\tau$, with τ the electronic elastic mean-free time. This leads to $\Gamma_z \sim \Gamma_\perp \sim (q/k_{\rm F})^2 v_{\rm F}^2\tau$.

Now consider the singularities at $T > 0$. In a solid-state system they are cut off at a length scale L that is inversely proportional to the spin-orbit coupling squared, or, since q is proportional to the spin-orbit coupling, $L \sim 2\pi k_{\rm F}/q^2$ [7]. The cutoff frequency for the dynamical singularities is given by $\Omega_0(k \approx 1/L)$. With these estimates we obtain

$$\frac{\delta\Gamma_z}{\Gamma_z} \sim \frac{T}{\epsilon_{\rm F}}\,\frac{\epsilon_{\rm F}}{\lambda}\,\frac{L}{\ell}\ , \tag{16a}$$

$$\frac{\delta\Gamma_\perp}{\Gamma_\perp} \sim 0.003\,\frac{T}{\epsilon_{\rm F}}\left(\frac{k_{\rm F}}{q}\right)^3 \frac{1}{(k_{\rm F}\ell)^2}\,\ln(k_{\rm F}L), \tag{16b}$$

with $\ell = v_{\rm F}\tau$ the elastic mean-free path. For the elastic constants we find

$$\frac{\delta c_z}{c_z} \sim -\frac{\delta c_\perp}{c_\perp} \sim -100\,\frac{T}{\epsilon_{\rm F}}\left(\frac{\epsilon_{\rm F}}{\lambda}\right)^2 \frac{q}{k_{\rm F}}\,\ln(k_{\rm F}L), \tag{17}$$

Parameter values for MnSi are ([7] and references therein) $\epsilon_{\rm F} \approx 23{,}000\,{\rm K}$, $q/k_{\rm F} \approx 0.024$, and $L \approx 7{,}500\,\AA$. The value of $\epsilon_{\rm F}/\lambda$ is less certain, we will assume $\epsilon_{\rm F}/\lambda = 4$, which leads to $\delta c_z/c_z \approx 0.1$ at $T = 10\,{\rm K}$. The corrections to the damping coefficients are much smaller. With a mean-free path $\ell \approx 5{,}000\,\AA$ [17] one finds $\delta\Gamma_z/\Gamma_z \approx 10^5\delta\Gamma_\perp/\Gamma_\perp \approx 10^{-3}$ at $T = 10\,{\rm K}$. In less clean samples the fluctuations of Γ_z and $\Gamma_\perp$ will obviously be larger.

The temperature scale where the divergent classical results cross over to the finite expressions in the quantum limit follows from Eqs. (13a,15), and is corroborated by an inspection of the relevant integrals. It is given by

$$T^* = \Omega_0(k_z = k_\perp = 1/L) \approx \gamma a^{1/2} q c_z^{1/2}/L. \tag{18}$$

With the same parameter values as above we find $T^* \approx 0.3\,{\rm mK}$. In interpreting all of these quantitative results one needs to keep in mind that these are *very* rough estimates that depend strongly on some of the parameters.

In conclusion, we find anomalously large fluctuation effects in the ordered phase of helimagnets which lead to a breakdown of hydrodynamics. In MnSi, the largest observable signature is predicted to be a linear-in-T shift in the resonance frequency squared, with an absolute value of the shift of about 10% at $T = 10\,{\rm K}$. This should be observable by neutron scattering.

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[11] In a perfectly clean system at zero temperature the transport coefficients are functions of frequency and wave number that diverge in the limit of zero frequency and wave number. We assume a small amount of quenched disorder to be present that makes these coefficients finite without having any other effects.

[12] Notice that, in the framework of flucuating hydrodynamics, $m(\boldsymbol{k},\omega)$ is c-number valued, and the relevant correlation function is therefore the fluctuation function or symmetrized correlation function F, which is an even function of the frequency, rather than the van Hove function.

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[15] aq^2, which is the coefficient of m^2 in Eq. (2), is not renormalized since we have omitted terms non-Gaussian in m from the action.

[16] In liquid crystals, a renormalization-group treatment resums the divergency in the analog of Eq. (14a) to a more complicated expression that goes to zero as $k_\perp \to 0$ [6], and the same is expected here.

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